Optimal Worksites on Highway Networks Subject to Constraints

Dr. Rade Hajdin & Dr. Bryan T. Adey
Outline

• Motivation
• Mathematical formulation
• Solution steps
• Example results
• Discussion of results
• Conclusions
Motivation

Parliamentary Committee - Problem

- Parliamentary Committee was invoked to investigate the preservation practice on Swiss National Highway System
- Increasing discontent with planning of work zones on National Highway Systems
- Short uncoordinated work zones
- Work zone induced congestions costs are estimated to be about 200 Mio. CHF (1999)
- Yearly preservation budget is ca. 155 Mio. CHF (1999)
Motivation

Parliamentary Committee - Measures

• Monetarisation of consequences for road users and at least theoretically for third parties
• Search for optimum balance between agency costs and user costs
• Swiss Federal Roads Office launched a project called UPlaNS to implement these measures
• 15 km / 50 km / 10 years strategy
• Integration of current BM, PM and EM practices
• Definition of 15 km corridors?
• Static or dynamic
Motivation

Static corridors

- Static corridors with predefined time slots for preservation actions
- Existing BMS and PMS suggest actions only in these time slots.
- Minimal adaptation of current BMS and PMS
- Easy to understand
- Suboptimal
Dynamic corridors

• Dynamic corridors are defined for each funding period (5 years; 5 to 10 years from present)
• Existing BMS and PMS suggest actions in funding period (optimum, minimum, do nothing).
• Optimum: Minimum long-term agency costs for a given traffic regime
• Corridor is sought that minimizes the sum of long-term agency and user costs
Motivation

Costs

• Long-term agency costs of a corridor = sum of long-term agency costs for actions suggested by BMS and PMS

• Traffic control costs; rough estimate of long-term traffic control costs
  – Fix sum and per unit length

• User costs; rough estimate of long term user costs
  – Fix sum and per unit length
  – Congestion
  – Accident
  – Operation
Introduction

Solution

• An algorithm developed to form an optimal worksite on a network, taking into consideration
  – agency costs associated with each individual intervention
  – Agency (traffic control costs and user costs attributable directly to the worksite, and
  – network connectivity.
Background

Network

Physical nodes of the highway network canton of Vaud, Switzerland
## Background

### Road links

<table>
<thead>
<tr>
<th>Section</th>
<th>Start node</th>
<th>End node</th>
<th>Length (km)</th>
<th>Number of nodes</th>
<th>Road sections where interventions are possible</th>
<th>Road sections where interventions are not possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>8</td>
<td>49</td>
<td>8</td>
<td>2-3, 4-5, 6-7</td>
<td>1-2, 3-4, 5-6, 7-8</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>11</td>
<td>6</td>
<td>4</td>
<td>9-10</td>
<td>8-9, 10-11</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>14</td>
<td>3</td>
<td>4</td>
<td>12-13</td>
<td>8-12, 13-14</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>23</td>
<td>27</td>
<td>10</td>
<td>15-16, 17-18, 19-20, 21-22</td>
<td>14-15, 16-17, 18-19, 20-21, 22-23</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
<td>26</td>
<td>6</td>
<td>4</td>
<td>24-25</td>
<td>23-24, 25-26</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>31</td>
<td>35</td>
<td>6</td>
<td>27-28, 29-30</td>
<td>23-27, 28-29, 30-31</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>38</td>
<td>21</td>
<td>8</td>
<td>32-33, 34-35, 36-37</td>
<td>14-32, 33-34, 35-36, 37-38</td>
</tr>
</tbody>
</table>
Background

Definitions

- Intervention-traffic regime = the specific combination of the type of intervention to be performed on a structure and the traffic regime required during the intervention.
- A link = part of the network comprised of all road sections between either intersections in the network, intersections and network limits, or between network limits, that consists of a single path.
Background

Constraints

• Maximum worksite length constraint
  – 25 km.

• Budget constraint, ranging from
  – “unlimited” (29.7 units), i.e. all possible interventions can be performed, to
  – “nothing” (3.9 units), at
  – intervals of 4.3 units.
Mathematical formulation

Directed model of network
Mathematical formulation

Objective function

Maximize \( Z = \sum_{(i,k)[j,l] \in A} y_{i,k}[j,l] \cdot \left( N_{i,k}[j,l] + C_{i,k}[j,l] \right) \)

where: \( C_{i,k}[j,l] = V_{i,k}[j,l] + V_{i,k}[j,l] + U_{i,k}[j,l] + U_{i,k}[j,l] \)

- is net agency benefit
- initial and subsequent traffic control costs
- initial and subsequent user costs
- a binary variable that takes the value of 1 if arc belongs to the path that maximizes \( Z \) and 0 if it does not
Mathematical formulation

Length constraint

\[
\sum_{[b,0]} y_{[c,l]} - \sum_{l} y_{[f,l][g,0]} \leq 0
\]

\(([c,l]):([b,0],[c,l]) \in A \land l \neq 0\)

for all \(s \in N\) and \(d_{[c,l][d,l]} \leq \Lambda < d_{[c,l][g,0]}\)
Mathematical formulation

Budget constraint

\[
\sum_{([i,k],[j,l]) \in A} y_{[i,k],[j,l]} \cdot C_{[i,k],[j,l]}^i \leq R
\]

where

- \( R \) is the maximum resources to be allocated.
Mathematical formulation

Continuity constraints

\[ 1 + \sum_{([j,l],[i,k]) \in A} y_{[j,l],[i,k]} = \sum_{([j,l],[a]) \in A} y_{[a],[j,l]} \]

\[ \sum_{([j,l],[e]) \in A} y_{[j,l],[e]} = 1 + \sum_{([j,l],[e,j,l]) \in A} y_{[e,j,l]} \]

\[ \sum_{([j,l],[i,k]) \in A} y_{[j,l],[i,k]} = \sum_{([j,l],[i,k]) \in A} y_{[i,k],[j,l]} \text{ for all } [i,k] \in (N \cap N_s) \wedge [i,k] \neq a \wedge [i,k] \neq e \]

\[ y_{[s],[j,l]} = \sum_{([j-1,k]=[j-1,k]) \in A} y_{[j-1,k],[j,l]} = 0 \text{ for all } [j,l] \in N_v \]

\[ y_{[j,l],[e]} = \sum_{([j-1,k]=[j-1,k]) \in A} y_{[j-1,k],[j,l]} = 0 \text{ for all } [j,l] \in N_z \]
Optimal worksites

Budget: 29.7 (273.4)
Optimal worksites

Budget: 25.4 (252.6)
Optimal worksites

Budget: 21.1 (232.2)
Optimal worksites

Budget: 16.8 (183.4)
Optimal worksites

Budget: 12.5 (138.6)
Optimal worksites

Budget: 8.2 (68.5)
Optimal worksites

Budget: 3.2 (0)
Discussion of results

• Optimal work site is centered on an intersection
Discussion of results

Intersection
Discussion of results

- Optimal work site is centered on an intersection
- Small reductions in budget can change the road sections to be included in the optimal worksite
# Discussion of results

## Road sections

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>29.7</td>
<td>273.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25.4</td>
<td>252.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21.1</td>
<td>232.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.8</td>
<td>183.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.5</td>
<td>138.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.2</td>
<td>68.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.9</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conclusions & next steps

• The presented algorithm is an efficient algorithm to be used to determine optimal worksites on a highway network.

• The next steps are to investigate
  – real world interventions
  – different numbers of intervention options
  – multiple worksites
  – optimal single worksites in time
  – optimal multiple worksites in time (Complexity!)