Spatial analysis of flood risk using coupled reliability and inundation models

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Theoretical basis

Consider a system consisting of $l$ basic variables $x := (x_1, \ldots, x_l)$ and limit state function $g(x)$. Damage, $D$, occurs when the system is in a ‘failed’ state. The risk, $R$, associated with the system is:

$$R = \int I(g(x) \leq 0) f(x) D \, dx$$
Theoretical basis

Consider a system consisting of \( l \) basic variables \( \mathbf{x} := (x_1, \ldots, x_l) \) and limit state function \( g(\mathbf{x}) \). Damage, \( D \), occurs when the system is in a ‘failed’ state. The risk, \( R \), associated with the system is:

\[
R = \int I(g(\mathbf{x}) \leq 0) f(\mathbf{x}) D d\mathbf{x}
\]

If the amount of damage will depend upon the values of \( \mathbf{x} \) when failure takes place, replace \( D \) with a function \( D(\mathbf{x}) \):

\[
D(\mathbf{x}) : \begin{cases} 
D(\mathbf{x}) = 0 & : g(\mathbf{x}) > 0 \\
D(\mathbf{x}) > 0 & : g(\mathbf{x}) \leq 0
\end{cases}
\]
Theoretical basis

Consider a system consisting of $l$ basic variables $\mathbf{x} := (x_1, \ldots, x_l)$ and limit state function $g(\mathbf{x})$. Damage, $D$, occurs when the system is in a ‘failed’ state. The risk, $R$, associated with the system is:

$$R = \int I(g(\mathbf{x}) \leq 0) f(\mathbf{x}) D d\mathbf{x}$$

If the amount of damage will depend upon the values of $\mathbf{x}$ when failure takes place, replace $D$ with a function $D(\mathbf{x})$:

$$D(\mathbf{x}) : \begin{cases} D(\mathbf{x}) = 0 & : g(\mathbf{x}) > 0 \\ D(\mathbf{x}) > 0 & : g(\mathbf{x}) \leq 0 \end{cases}$$

The risk calculation is therefore:

$$R = \int f(\mathbf{x}) D(\mathbf{x}) d\mathbf{x}$$
Fluvial flooding system: Burton-upon-Trent, England
Typical flood outlines
The risk calculation

\[ R = \int \sum_{j=1}^{2^n} P(S_j \mid Q) f(Q) D(Q, S_j) dQ \]

where

- \( R \) is the total risk
- \( Q \) is the river flow at the upstream boundary of the model
- \( S_j; j = 1, \ldots, 2^n \) is the dike system state
- \( D(Q, S_j) \) is the flood damage, given discharge \( Q \) and system state \( S_j \)

\( P(S_j \mid Q) \) is calculated from:
- a hydrodynamic model, that calculates the water level at each dike, given \( Q \), and
- a reliability analysis of each dike section, assuming independent dike response given load \( Q \)
Initial sampling of load $Q$ and system states $S_j$
Risk estimate (UK £1,000s)

No. simulations

Risk estimate (UK £1,000s)

No pre-selection
Pre-selection from $P(S_j|Q)$
Pre-selection from $D(Q,S_j)P(S_j|Q)$
\( P(S|Q)f(Q)D(Q,S) \)
Spatial distribution of flood risk and contribution of each dyke to this flood risk.
Spatial distribution of annual inundation probability
Failure probability (solid line) and contribution towards flood risk (dashed line) of each dike section
Coastal flooding system: Towyn, Wales
Typical flood outlines

Joint loading variables, \( v := (H_s, W) \)

Dike L fails: \( D(S_J, v) = £43\text{million} \)

Dike 4 fails: \( D(S_J, v) = £79\text{million} \)

Dike I fails: \( D(S_J, v) = £106\text{million} \)

Dike C fails: \( D(S_J, v) = £41\text{million} \)
Failure modes of dike sections, conditional upon $H_s$ and $W$

- Dike I – Shingle beach erosion failure
- Dike K – Dune erosion failure
- Dike D – Rock armour failure
- Dike 4 – Piping failure
Relationship between risk and failure probability
Spatial distribution of annual flood probability
Spatial distribution of flood risk
Conclusions
Flood risk is a complex function of:
- The severity of the (often multi-variate) loading
- The reliability of the flood defence systems
- The floodplain topography
- The location of people and assets in the floodplain

For complex topographies, hydrodynamic inundation modelling is necessary to understand the spatial distribution of risk

Importance sampling methods developed for reliability analysis are not necessarily efficient for risk analysis where the consequences calculation is computationally expensive
References


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