Cost-benefit Importance Vectors for Design Improvement Decisions

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Abstract

The primary objective in this paper is to put forward importance vectors that identify the structural parameters that are most cost-efficient to change to improve the reliability of the design. The methodology comprises an amalgamation of reliability sensitivity measures and construction cost data. The resulting importance vectors expose the change in the reliability per dollar spent to change each parameter. A novel distinction is made between design parameters and tolerance parameters in order to include the possibility of improving the reliability by allocating resources to reduce variability in certain geometry and material parameters. The methodology utilizes sophisticated structural models in conjunction with state-of-the-art reliability methods. In this paper the incremental costs are estimated based on RSMeans construction cost data, while the probabilistic models for material and geometry parameters are based on the JCSS model code. A detailed example of a six-storey, three-bay reinforced concrete frame structure is presented to demonstrate the computation and interpretation of the cost-benefit importance vectors.

1. Introduction

Importance vectors are put forward in this paper to guide the engineer who is faced with the task of improving an under-performing structural design. The costs of changing parameter values, as well as the cost of prescribing tighter construction tolerances are considered. Notably, the importance vectors are developed within a structural reliability framework in order to identify which parameters should be changed to improve the balance between cost and safety. The developments build upon the “finite element reliability” methodology described in, e.g., Haukaas and Der Kiureghian (2004). In this approach, parameters that represent material properties, cross-sectional geometry, nodal coordinates and loads are characterized as random variables. Subsequently, the probability of exceeding specific structural response thresholds is computed. Of particular significance to the present study is the availability of sensitivity and importance measures from reliability analysis (Hohenbichler and Rackwitz 1986, Bjerager and Krenk 1989, Haukaas and Der Kiureghian 2005).

In this paper it is recognized that the traditional reliability importance measures do not take into account the cost of modifying a parameter. In fact, parameters that appear important may be too costly to modify. This motivates the development of importance measures that identify the parameters that are most cost-efficient to change to improve a design. In addition to guiding the engineer in the process of design improvement, the paper addresses the decision making regarding prescription of tolerances and inspection in the construction phase. Inspection is devised to ensure that the variability in geometry parameters and some material parameters stay within prescribed tolerance limits. It is of interest to identify the parameters for which such inspection efforts should
be focused. Conversely, resources may be saved by identifying the parameters for which a change in the variance has insignificant effect on the reliability.

2. Identification of Design Parameters and Tolerance Parameters

The feasibility of altering a structural parameter may seem different from the viewpoint of an analyst – particularly an academic analyst – compared to that of a design engineer. A parameter that is identified as important by means of response sensitivities or reliability importance measures may be costly or impractical to modify. Implicit in an efficient design is the engineer’s awareness of the feasibility, fabrication cost, and construction cost of different alternatives. A key objective in this paper is to merge this type of information with sophisticated analysis to obtain useful parameter rankings.

The first step towards functional importance measures is the classification of parameters in the structural analysis model from a practical design perspective. Given an under-performing trial design, two questions are of importance. The first is which characteristic design values can be modified to improve the structural performance, albeit at a cost. Due to the presence of uncertainty it is inappropriate to consider the outcome of a parameter as a prescribed design value. Indeed, the outcome is uncertain. Instead, it is a characteristic value of the probability distribution that must be considered. For example, an increase in the mean or the 95-percentile steel strength may be prescribed by the engineer to improve the performance. This type of characteristic parameters are termed design parameters and collected in the vector $d$. The second question is which sources of variability that can be reduced to improve the reliability. This question recognizes that variability in certain parameters is affected by tighter tolerances and more rigorous inspection schemes. For example, the variability in concrete strength may be influenced by the manufacturer through improved precision in the mixing process. Another example is geometrical imperfections. Such parameters are here termed tolerance parameters and collected in the vector $t$. In the following, cost-benefit importance measures are developed for these classes of parameters.

3. Reliability Sensitivities

A key ingredient in the cost-benefit ranking of structural parameters is sensitivity information. Specifically, it is the sensitivity of the reliability with respect to changes in the parameters $d$ and $t$ that is of interest. Consequently, two sensitivity measures are considered; the sensitivity of the reliability with respect to the mean of the random variables and the sensitivity of the reliability with respect to their standard deviation. In the first-order reliability method (FORM), which has been sucessfully applied in finite element reliability analysis with several hundred random variables and nonlinear structural response (Haukaas and Der Kiureghian 2004), the derivative of the reliability index with respect to the mean, $\mu_i$, of random variable number $i$ reads

$$\frac{\partial \beta}{\partial \mu_i} = \frac{\partial \beta}{\partial \mu^*_i} \frac{\partial \mu^*_i}{\partial \mu_i}$$

(1)

where $u$ is the vector of random variables in the standard normal space, $u^*$ is the point of linearization in FORM, $\partial \beta/\partial u^* = u^*/\| u^* \|$, and $\partial u^*/\partial \mu_i$ is obtained by differentiating the probability transformation between the $u$-space and the original $x$-space. By applying the same procedure to determine reliability sensitivities with respect to the standard deviation of the random variables, the following two reliability sensitivity vectors are available from the FORM reliability analysis:
\[ \frac{\partial \beta}{\partial \mu_d} = \frac{\partial \beta}{\partial \hat{u}}^T \frac{\partial \hat{u}}{\partial \mu_d} \] and \[ \frac{\partial \beta}{\partial \sigma_t} = \frac{\partial \beta}{\partial \hat{u}}^T \frac{\partial \hat{u}}{\partial \sigma_t} \] (2)

where \( \mu_d \) and \( \sigma_t \) are the vectors of means and standard deviations of the parameters in the vectors \( d \) and \( t \), respectively. In the following, these sensitivity vectors are utilized in the development of cost-benefit importance measures. It is noted that the components of the vector \( \frac{\partial \beta}{\partial \mu_d} \) cannot be compared due to differing units. The same is the case for the elements of \( \frac{\partial \beta}{\partial \sigma_t} \). Although several importance vectors with uniform dimensions are available from reliability theory they do not contain information about the feasibility and cost of modifying the parameters.

4. Incremental Cost Information

Traditionally, design of structures is subject to experience and informal information, as much as mechanics and structural analysis. The experienced engineer is aware of cost and feasibility associated with increasing the column dimensions versus increasing the amount of steel reinforcement, etc. Although such information may seem indefinable to an inexperienced engineer, much of this information can be formalized. For example, the opinion that it would be more expensive to increase the amount of reinforcement can be founded on construction cost data. Conversely, some opinions of the experienced engineer might not be tangible but due to convenience and tradition. Such information might not be as valuable, because better alternatives than the customary solution may be available. The formalization of tangible cost information associated with modifying an underperforming design is the subject of this section.

The building construction cost data published by RSMeans (2000) is selected as the key cost reference in this study. In essence, this is a price guide for the construction industry on the North-American continent. It provides unit costs of most constituents in modern construction, ranging from concrete and reinforcement steel to finishes and furnishings. It is recognized that the engineer on a particular project will have even more information at his/her disposal. For example, the concrete manufacturers in a particular region may be able to provide information about the cost associated with reducing the variability in the concrete strength. However, such information is usually not published in referable formats. The garnering and publishing of such information would be a useful task for future studies, although outside the scope of this paper. The design engineer may also be aware of particular constraints, such as preferences from the architect or developer, or the availability of certain steel profiles, say, in certain regions of the market. However, the task of optimizing the design within constraints can only be solved by reliability-based design optimization algorithms that are computationally more costly and complex than the practical methodology put forward in this paper.

According to the previous classification of design and tolerance parameters, two categories of incremental costs are sought: the incremental cost associated with changing the mean of selected parameters and the incremental cost associated with changing the variability (standard deviation) of selected parameters. This incremental cost information is collected in the vectors

\[ \frac{\partial C}{\partial \mu_d} = \text{change in cost due to a unit change in mean values} \] (3)

and
\[
\frac{\partial c}{\partial \sigma_i} = \text{change in cost due to a unit change in standard deviations} \quad (4)
\]

where \( c \) is the construction cost and the vectors \( \mu_d \) and \( \sigma_i \) correspond to those in Eq. (2).

For many of the parameters relevant to this study the RSMeans (2000) handbook conveniently provides costs per unit volume or weight. However, for some parameters the cost data is provided in a discrete format. For instance, the steel strength must be specified as an available strength class; no continuous change in the strength is possible. Reinforcing steel, for example, are available in normal and high-strength alternatives. Under such circumstances the incremental cost measures in Eqs. (3) and (4) may be determined by interpolation. Although approximate, this approach is in this study justified by our primary objective; namely, to provide an indication of which parameters should be addressed from a cost-benefit standpoint. Only after critical inspection of the modified design can the engineer determine whether the suggested changes are feasible and efficient. Detailed incremental cost information in the format of Eqs. (3) and (4) is provided in the subsequent numerical example.

5. Ranking of Design Parameters

As mentioned earlier, the engineer can – from a realistic standpoint – not specify the outcome of a structural parameter. Instead, the engineer specifies characteristics of the probability distribution. For example, when a steel strength of 500 N/mm\(^2\) is specified it is not the outcome but rather a characteristic value that is given. The same is the case for other material parameters, nodal coordinates, structural dimensions, reinforcement placement, and geometrical imperfections. This motivates the selection of mean values as design parameters. To rank these parameters according to a cost-benefit criterion the first reliability sensitivity measure in Eq. (2) is employed in conjunction with the incremental cost information in the format given in Eq. (3). The importance vector is defined as

\[
\delta_d = \left( \frac{\partial \beta}{\partial \mu_d} \cdot \frac{\partial \mu_d}{\partial c} \right) \quad (5)
\]

where \( \langle a \cdot b \rangle \equiv a_i b_i \) (no summation implied) denotes a vector with components equal to the product of the corresponding components of vectors \( a \) and \( b \). The quantity \( \frac{\partial \mu_d}{\partial c} \) in Eq. (5) is a vector with components \( (\partial c/\partial \mu_i)^{-1} \), that is, the inverse of the components of the vector in Eq. (3). Important, the components of the vector \( \delta_d \) in Eq. (5) are interpreted as \( \frac{\partial \beta}{\partial c} \), that is, the cost associated with changing the reliability, for each design parameter. The parameter with highest \( \frac{\partial \beta}{\partial c} \)-value is the most cost-efficient design parameter to address to improve the reliability of the design.

6. Ranking of Tolerance Parameters

The present study includes the cost associated with inspection to reduce variability in certain parameters. In particular, previous finite element reliability studies have shown that variability in geometry parameters may have significant impact on structural reliability assessments (Haukaas
and Der Kiureghian 2005). Different countries have different inspection practices to deal with this source of uncertainty. Clearly, increased inspection carries increased construction costs, at the benefit of reduced uncertainty. Hence, it is in this study considered possible to influence the variance of the probability distributions for specific geometry and material parameters. Although limited formal cost data is available for inspection specifications, approximate cost assumptions are made in this paper. Future studies are encouraged to gather this type of cost data.

Analogous to the treatment of design parameters, Eqs. (2b) and (4) are employed to define the cost-benefit importance vector for tolerance parameters:

$$\mathbf{\delta}_t = \left[ \frac{\partial \beta}{\partial \sigma_t},\frac{\partial \sigma_t}{\partial c} \right]$$

(6)

The components of the vector $\mathbf{\delta}_t$ are interpreted as $\partial \beta / \partial c$, that is, the cost of changing the reliability for each tolerance parameter. The parameter with highest $\partial \beta / \partial c$-value is the most cost-efficient parameter to address to improve the reliability.

7. Numerical Example

A six-storey reinforced concrete frame structure is selected as an example to demonstrate the computation and interpretation of the vectors $\mathbf{\delta}_d$ and $\mathbf{\delta}_t$. The initial design for the structure is taken from the seismic design section of the Canadian Concrete Design Handbook (Cement Association of Canada 1995). This reference provides complete design specifications including member dimensions and amount of reinforcement. It is stressed that our utilization of importance vectors to identify the parameters that should be changed to improve the reliability by no means implies a criticism of the proposed design. To emphasize this point, the methodology is demonstrated by considering only one of the bays for seismic loading. The building has seven bays with 6m spacing in the North-South (NS) direction and three bays (two office bays with 9m spacing and a central 6m corridor bay) in the East-West (EW) direction. All interior columns are 500 by 500mm, while all exterior columns are 450 by 450mm. The beams of both NS and EW frames are 400mm wide by 600mm deep for the first three storeys and 400 by 550mm for the top three storeys. In the analysis model, each column is reinforced with 12 No. 25 longitudinal bars evenly distributed. Each beam is modeled with 4 No. 20 longitudinal bars both at the top and at the bottom of the cross-section.

In this example a 2D nonlinear static pushover analysis of the second EW frame is considered. The finite element model is shown in Fig. 1. The load case of “(1.0)(dead load) + (0.5)(live load) + (1.0)(earthquake load)” is considered in the analysis. The total lateral loads in the static pushover reliability analysis, including the effect of torsion, are considered as lognormal random variables with means from the handbook ($H_1=25.9kN$, $H_2=44.5kN$, $H_3=63.7kN$, $H_4=81kN$, $H_5=99.8kN$, $H_6=119.9kN$), 15% coefficient of variation, and correlation coefficients equal to 0.8. The gravity loads are deterministic with values $P_1=108kN$, $P_2=105kN$, $P_3=96kN$, $P_4=184kN$, $P_5=178kN$, $P_6=182kN$.

Figure 1: Analysis model of reinforced concrete frame. Element numbers are in circles.

The concrete design handbook does not contain specific information about the probability distribution for the structural parameters. For this purpose the JCSS model code (2001) is employed, which contains probabilistic models for the concrete and reinforcing steel properties and the geometrical imperfections. The modulus of elasticity of the reinforcing steel is assumed to
be deterministic \( E=205 \text{GPa} \). The standard deviation of the yield stress is 30MPa. Because the nominal yield stress of the reinforcing steel is 400MPa the mean yield stress is \((400) + (2)(30) = 460 \text{MPa}\). This implies a coefficient of variation of 6.5%. In the analysis, the cross-sections are discretized into 20 fibers in the in-plane direction; thus allowing uniaxial material models to be applied. The reinforcing steel is modeled with a smoothed bi-linear material model with hardening modulus 5% of \( E \). The smoothing of the transition between the elastic and plastic response regimes takes place at 80% of the yield stress to avoid gradient discontinuities in the reliability analysis.

The concrete fibers of the cross-sections are modeled by the elastic-perfectly-plastic material model. In this application this model is not smoothed because the high number of concrete fibers in the cross-section provides the desired smoothing effect. Based on the C25 concrete quality (JCSS 2001) the mean and the coefficient of variation of the concrete strength are computed to be 26.9MPa and 17%, respectively. The corresponding mean modulus of elasticity is computed to be 18.5GPa, with a coefficient of variation of 16%. The correlation coefficient between concrete strength and stiffness is computed to be approximately -0.6 (minus sign because the strength is negative and the stiffness is positive), based on a first-order, second-moment approximation applied to the probabilistic models.

The mean cover concrete depth is taken as 62.5mm throughout the structure, with a 10mm standard deviation based on interpretation of the JCSS model code. The mean of the outer dimensions of the columns and girders are kept at their nominal values provided in Fig. 1, while the standard deviations are assumed to be 7mm. The area of the steel reinforcement in each member is also considered uncertain, albeit with insignificant coefficient of variation, to include the mean reinforcement area in the cost-benefit importance ranking.

The above parameters are modeled by one random variable per member. In addition to the correlation between concrete strength and stiffness noted above, all reinforcement yield stresses are inter-correlated by 0.6 and the concrete strengths of the columns/girders in each storey are inter-correlated by 0.6. This is done with the assumption that each storey is constructed from one batch of concrete. In passing it is noted that care must be exercised when specifying correlation to avoid a singular correlation matrix.

The nodal coordinates are considered to be random due to uncertain geometrical imperfections and eccentricities. Interpretation of the JCSS model code leads to the selection of 7mm as standard deviation for the horizontal coordinates and 5mm as the standard deviation for the vertical coordinates. In total, 356 random variables are present.

The pushover analysis is performed with the OpenSees software (http://opensees.berkeley.edu) with one fiber-discretized force-based element per member. OpenSees is an open-source, object-oriented finite element software developed specifically for earthquake engineering. It has previously been extended with reliability, response sensitivity, and design optimization features (Haukaas and Der Kiureghian 2004, Liang et al. 2006). In this study OpenSees is employed to carry out FORM reliability analysis for the finite element model presented above. The failure event is considered to be the roof drift exceeding 2%. Consequently, the following limit-state function is prescribed:

\[
g(x) = (23.1)(0.02) - r_{\text{roof}} (7)
\]

where 23.1m is the height of the frame and \( r_{\text{roof}} \) is the roof displacement. The FORM reliability analysis results in the reliability index \( \beta=3.93 \) and corresponding failure probability \( p_f = 4.3 \times 10^{-5} \).
The two reliability sensitivity vectors that are defined in Eq. (2) are also computed. To post-process these two vectors the classification of design and tolerance parameters that is outlined previously is adopted. Next, approximate incremental cost associated with the modification of each parameter is obtained from the RSMeans handbook and from judgment. In summary, the following cost-assumptions are made:

- Incremental cost of concrete: $93 per m³ (excluding labor and equipment).
- Incremental cost of steel reinforcement: $0.615 per kg (excluding labor and equipment).
- Incremental cost of increasing the mean concrete strength: $1.4 per MPa and m³. (This is based on interpolation between relevant concrete qualities. It is noted that this incremental cost increases to about $5 per MPa and m³ for high-strength concretes.)
- Incremental cost of increasing the mean concrete stiffness: $0.0043 per MPa and m³. (This cost is directly related to the incremental cost in the previous item due to the utilized functional relationship between concrete strength and stiffness.)
- Incremental cost of reducing the standard deviation of the concrete strength by 1MPa for each column is assumed to be the same as the cost of concrete for that column. (This cost is based on rough estimates to demonstrate the methodology. It is envisioned that concrete manufacturers may provide estimates for this cost.)
- Incremental cost of increasing the mean steel strength is assumed to be $0.33 \times 10^{-3} per MPa and kg. (This is based on linear interpolation between normal-strength and high-strength steel to get an indication of the importance of the steel strength.)
- Incremental cost of reducing the standard deviation of the geometrical imperfection in structural dimensions and cover depth is assumed to be $100 per mm for each structural member. (This assessment is based on rough estimates to demonstrate the methodology and to point out the value of gathering this type of cost information.)

The incremental cost values above are multiplied by relevant volumes and weights of each member to obtain the incremental cost for each parameter. The resulting values are merged with the reliability sensitivities to obtain the results in Tables 1 and 2. Table 1 shows the 15 highest ranked design parameters, while Table 2 shows the 15 highest ranked tolerance parameters. Interestingly, there is orders of magnitude difference between the $\partial \beta / \partial c$-values of the top ranked design parameter and the top ranked tolerance variable. This indicates that it is significantly more cost-efficient to change the design parameters than to allocate resources for inspection and tighter tolerances to reduce the variability as a means of improving the reliability. Table 1 shows that the horizontal coordinate of the rightmost base nodes and the yield strength of the reinforcement steel of the corridor-span girders 26, 29, and 32 are the most efficient design parameters to change to improve the reliability. However, from a practical perspective it is not useful to perform this study for individual nodes because this would make the structural geometry unappealing. This argument also applies to the cross-sectional dimensions of the columns; the first ranks as number 18. Furthermore, the engineer will likely argue that it is undesirable to change the yield strength by procuring high-strength steel. Therefore, a study is carried out in which the yield strength is not considered as a design parameter and where only a joint change in geometrical parameters is accepted. For example, the cost-benefit ranking of changing the horizontal coordinates of all the nodes in the rightmost wall is studied (effectively making the right girder span wider), as well as enforcing equal cross-section changes for all columns that are initially the same, within a storey. These results are obtained by adding the relevant sensitivity results and incremental costs individually before evaluating Eq. (5). The following parameters then emerge as most important ($\partial \beta / \partial c$-values in parenthesis): width of columns 7 & 13 (7.25 \times 10^{-3}), width of columns 1 & 19 (7.11 \times 10^{-3}), horizontal location of rightmost wall (6.67 \times 10^{-3}), concrete strength of element 19 (3.61 \times 10^{-3}), concrete strength of element 13 (3.46 \times 10^{-3}), and concrete strength of element 7 (1.67 \times 10^{-3}).
It is interesting to observe that the elements that here are identified as important seem important also from an engineering judgment standpoint. For example, columns 1, 7, 13, and 19 are the columns at the base, which are expected to be important for the performance of the structure in the selected limit-state.

If resources are to be spent on reducing variability, then Table 2 indicates that it is the concrete strength of the lower region of the building that should be addressed. In fact, five of the highest ranking tolerance parameters are standard deviations of the concrete strength. In the next positions follows the standard deviation of the cover concrete depth of a number of members. However, the interpretation of the cover depth results must be made with caution. In the analysis model the cover depth is defined as the distance from the outer concrete surface to the outer edge of the reinforcement, with the outer dimensions of the cross-sections being determined by parameters $b$ and $h$. Effectively, an increase in the cover depth does not affect the outer dimensions, but rather pushes the reinforcement towards the center of the cross-section. This is the reason why the reliability sensitivity with respect to standard deviations of cover concrete depths have the opposite sign compared to other parameters. It is not obvious that this is what occurs in reality when the cover concrete depth varies. For this reason it may rather be of interest to observe that the further ranking of tolerance parameters contains a mix of concrete strengths and in-plane cross-sectional dimensions. That is, variability in these parameters is efficient to address by tighter tolerances and increased inspection. It is reemphasized, however, that the $\delta_t$-values are orders of magnitude lower than the $\delta_d$-values in this example.

### Table 1: Ranking of design parameters. Element numbers are shown in Fig. 1. Node numbers 1, 8, 15, and 22 are the base nodes from left to right. $\sigma_y$ is the mean strength of the reinforcement.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Node/element</th>
<th>Mean Sensitivity $\left(\frac{\partial \beta}{\partial \mu_d}\right)$</th>
<th>Incremental Cost $\left(\frac{\partial c}{\partial \mu_d}\right)$</th>
<th>Change in $\beta$ per dollar $\left(\delta_d\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x$-coord</td>
<td>15</td>
<td>1.04</td>
<td>37.7</td>
</tr>
<tr>
<td>2</td>
<td>$x$-coord</td>
<td>22</td>
<td>0.86</td>
<td>37.7</td>
</tr>
<tr>
<td>3</td>
<td>$\sigma_y$</td>
<td>26</td>
<td>$6.59 \times 10^{-10}$</td>
<td>$3.73 \times 10^{-8}$</td>
</tr>
<tr>
<td>4</td>
<td>$\sigma_y$</td>
<td>29</td>
<td>$6.55 \times 10^{-10}$</td>
<td>$3.73 \times 10^{-8}$</td>
</tr>
<tr>
<td>5</td>
<td>$\sigma_y$</td>
<td>32</td>
<td>$5.67 \times 10^{-10}$</td>
<td>$3.73 \times 10^{-8}$</td>
</tr>
<tr>
<td>6</td>
<td>$\sigma_y$</td>
<td>13</td>
<td>$5.50 \times 10^{-10}$</td>
<td>$3.77 \times 10^{-8}$</td>
</tr>
<tr>
<td>7</td>
<td>$\sigma_y$</td>
<td>19</td>
<td>$5.24 \times 10^{-10}$</td>
<td>$3.77 \times 10^{-8}$</td>
</tr>
<tr>
<td>8</td>
<td>$x$-coord</td>
<td>8</td>
<td>0.51</td>
<td>37.7</td>
</tr>
<tr>
<td>9</td>
<td>$\sigma_y$</td>
<td>27</td>
<td>$7.16 \times 10^{-10}$</td>
<td>$5.60 \times 10^{-8}$</td>
</tr>
<tr>
<td>10</td>
<td>$\sigma_y$</td>
<td>7</td>
<td>$4.82 \times 10^{-10}$</td>
<td>$3.77 \times 10^{-8}$</td>
</tr>
<tr>
<td>11</td>
<td>$\sigma_y$</td>
<td>28</td>
<td>$6.71 \times 10^{-10}$</td>
<td>$5.60 \times 10^{-8}$</td>
</tr>
<tr>
<td>12</td>
<td>$\sigma_y$</td>
<td>25</td>
<td>$6.66 \times 10^{-10}$</td>
<td>$5.60 \times 10^{-8}$</td>
</tr>
<tr>
<td>13</td>
<td>$\sigma_y$</td>
<td>30</td>
<td>$6.62 \times 10^{-10}$</td>
<td>$5.60 \times 10^{-8}$</td>
</tr>
<tr>
<td>14</td>
<td>$\sigma_y$</td>
<td>31</td>
<td>$5.96 \times 10^{-10}$</td>
<td>$5.60 \times 10^{-8}$</td>
</tr>
<tr>
<td>15</td>
<td>$\sigma_y$</td>
<td>35</td>
<td>$3.91 \times 10^{-10}$</td>
<td>$3.73 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

### Table 2: Ranking of tolerance parameters. Element numbers are shown in Fig. 1. $f'_{c}$ is standard deviation of the concrete strength, $c$ is the standard dev. of the cover concrete depth.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Element number</th>
<th>Stdv. Sensitivity $\left(\frac{\partial \beta}{\partial \sigma_t}\right)$</th>
<th>Incremental Cost $\left(\frac{\partial c}{\partial \sigma_t}\right)$</th>
<th>Change in $\beta$ per dollar $\left(\delta_t\right)$</th>
</tr>
</thead>
</table>


Conclusions

This paper puts forward two importance vectors to rank structural parameters according to which would be most efficient to change to improve the reliability of a design. Reliability sensitivity results from FORM reliability analysis are merged with incremental cost data. The result is vectors that reveal the change in reliability per dollar spent, for a range of parameters. A novelty in this study is the inclusion of uncertain geometrical imperfections and allocation of resources for inspection. As a demonstration, a six-storey, three-bay reinforced concrete frame is analyzed for reliability. The resulting cost-benefit importance vectors show that it is the geometrical dimensions of specific columns that are most efficient to increase to improve the reliability. If resources are to be spent on reducing variance then it turns out to be most efficient to reduce the variability in the concrete strength, for the example considered.

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