Influence of Correlation on Loss Estimation of Spatially Distributed Networks

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Motivation

• Limitations of prior research in seismic risk for a transportation network:
  – uncertainty around the loss decision variable
  – spatial correlation of ground motion
  – bridge structure-to-structure damage correlation

• Understanding the interrelatedness & uncertainty of components affects decision making in:
  – Design & Urban planning
  – Seismic Retrofit Prioritization
  – Emergency Response efforts
Objective

- Quantify uncertainty in component loss ($\sigma_L$) for a spatially distributed transportation network
  - Correlation of Ground Motion Residuals
    - Non-distance Dependent
    - Distance Dependent
  - Damage Correlation effects
- Sensitivity of $\sigma_L$ to both contributors of correlation

\[
\sigma_{L_{total}}^2 = \sum_{i=1}^{N} a_i^2 \cdot \sigma_{L_i}^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} a_i \cdot a_j \cdot \rho_{L_iL_j} \sigma_{L_i} \sigma_{L_j}
\]

- Bounds on CoV$_L$ for network physical loss
- Effect of network size on CoV$_L$
Independent Components

- Treatment of Uncertainties in Ground Motion and Damage for univariate case:

\[
E[L_i] = \sum_k \int E[L_i | D_i = d_i^{(k)}] \cdot P[D_i = d_i^{(k)} | U_i] \cdot f_{U_i} du_i dd_i
\]

\[
\sigma_{L_i}^2 = \sum_k \int E[L_i^2 | D_i = d_i^{(k)}] \cdot P[D_i = d_i^{(k)} | U_i] \cdot f_{U_i} du_i dd_i
\]

\[
- \left[ \sum_k \int E[L_i | D_i = d_i^{(k)}] \cdot P[D_i = d_i^{(k)} | U_i] \cdot f_{U_i} du_i dd_i \right]^2
\]

- Define Site Ground Motion Distribution
- Define Site Damage Distribution
Ground Motion at Single Site

- Variability of Ground Motion Amplitude:

\[ \ln U = f(M_w, R, V_s) + \varepsilon_{\ln U} \]

- Based on Attenuation Models for the Western United States
Damage State at Single Site

<table>
<thead>
<tr>
<th>Damage State</th>
<th>$\lambda$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slight</td>
<td>0.8000</td>
<td>0.6</td>
</tr>
<tr>
<td>Moderate</td>
<td>0.9973</td>
<td>0.6</td>
</tr>
<tr>
<td>Major</td>
<td>1.1967</td>
<td>0.6</td>
</tr>
<tr>
<td>Complete</td>
<td>1.6953</td>
<td>0.6</td>
</tr>
</tbody>
</table>

\[ P[D = d^{(k)}|U] = \Phi\left(\frac{\ln u - \ln \lambda}{\zeta}\right)_k - \Phi\left(\frac{\ln u - \ln \lambda}{\zeta}\right)_{k+1} \]
Dependent Components

• Treatment of Uncertainties in Ground Motion and Damage for multivariate case:

\[
\sigma_{L_iL_j} = \sum_{k_i=1}^{5} \sum_{k_j=1}^{5} \int \int E[L_i|D_i = d_i^{(k_i)}] \cdot E[L_i|D_j = d_i^{(k_i)}] \cdot P[D_i = d_i^{(k_i)}, D_j = d_j^{(k_j)}|U_i, U_j] \cdot f_{U_i,U_j}(u_i, u_j) du_i du_j dd_i dd_j \\
- \left[ \sum_{k_i=1}^{5} \int E[L_i|D_i = d_i^{(k_i)}] \cdot P[D_i = d_i^{(k_i)}|U_i] \cdot f_{U_i}(u_i) du_i dd_i \right] \\
\times \left[ \sum_{k_j=1}^{5} \int E[L_j|D_j = d_j^{(k_j)}] \cdot P[D_j = d_j^{(k_j)}|U_j] \cdot f_{U_j}(u_j) du_j dd_j \right]
\]

• Define Multi-Site Ground Motion Distribution
• Define Multi-Site Damage Distribution
Ground Motion at Multiple Sites

Joint Exceedence Surface for two sites
(Partially Correlated)

Isotropic Correlation Model:

\[ \rho_{i,j} = \frac{\sigma_e^2 + \exp\left\{ -\left( \frac{r_{ij}}{r_o} \right)^2 \right\} \sigma_s^2}{\sigma_e^2 + \sigma_r^2 + \sigma_s^2} \]
Damage State at Multiple Sites

- Joint discrete probability distribution for cases of non-zero & non-unity correlation?

\[
\min_P \left( \sum_{i,j} i \cdot j \cdot P_{ij} \right)^2 - \rho \left( \frac{\sum_{i,j} \sigma_{D_i|U_i} \sigma_{D_j|U_j} - E[D_i|U_i] \cdot E[D_j|U_j]}{\sigma_{D_i|U_i} \sigma_{D_j|U_j}} - \rho \right)
\]

\[
0 \leq P_{ij} \leq 1 \text{ for } \forall \ i, j \in \{D_i, D_j\}
\]

\[
P[D_i = d_i^{(k_i)}] = \sum_j P_{ij}
\]

\[
P[D_j = d_j^{(k_j)}] = \sum_i P_{ij}
\]
Damage State at Multiple Sites

\[ P[D_1 = d_i, D_2 = d_j | S_{a1}, S_{a2}] \]
Dependent Loss Estimation

• Computational expense of analytical solution → impractical for large networks

• Monte Carlo (MC) simulation:
  – Efficiently solves integrals
  – Sample correlated ground motions
  – Sample correlated discrete damage states conditioned on ground motion
  – Evaluate $E[L]$ and $\text{Var}[L]$
Selection of Scenario for Seismic Risk Analysis in Application

Table 6.4. Probabilities for the San Andreas fault, 2002–

<table>
<thead>
<tr>
<th>San Andreas Fault</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean [ 2.5% – 97.5% ]</td>
</tr>
<tr>
<td>Entire Fault system</td>
<td></td>
</tr>
<tr>
<td>All ruptures</td>
<td>0.238 [ 0.029 – 0.531 ]</td>
</tr>
<tr>
<td>Ruptures M≥6.7</td>
<td>0.236 [ 0.029 – 0.524 ]</td>
</tr>
<tr>
<td>Ruptures M≥7.0</td>
<td>0.182 [ 0.015 – 0.379 ]</td>
</tr>
<tr>
<td>Ruptures M≥7.5</td>
<td>0.090 [ 0.008 – 0.189 ]</td>
</tr>
<tr>
<td>Fault segments - All ruptures</td>
<td></td>
</tr>
<tr>
<td>SAS</td>
<td>0.113 [ 0.010 – 0.238 ]</td>
</tr>
<tr>
<td>SAP</td>
<td>0.133 [ 0.010 – 0.295 ]</td>
</tr>
<tr>
<td>SAN</td>
<td>0.116 [ 0.014 – 0.235 ]</td>
</tr>
<tr>
<td>SAO</td>
<td>0.107 [ 0.011 – 0.220 ]</td>
</tr>
<tr>
<td>Rupture sources (Mean magnitude)</td>
<td></td>
</tr>
<tr>
<td>SAS (7.03)</td>
<td>0.026 [ 0.000 – 0.108 ]</td>
</tr>
<tr>
<td>SAP (7.15)</td>
<td>0.044 [ 0.000 – 0.172 ]</td>
</tr>
<tr>
<td>SAN (7.45)</td>
<td>0.009 [ 0.000 – 0.037 ]</td>
</tr>
<tr>
<td>SAO (7.29)</td>
<td>0.009 [ 0.000 – 0.043 ]</td>
</tr>
<tr>
<td>SAS+SAP (7.42)</td>
<td>0.035 [ 0.001 – 0.102 ]</td>
</tr>
<tr>
<td>SAP+SAN (7.65)</td>
<td>0.000 [ 0.000 – 0.000 ]</td>
</tr>
<tr>
<td>SAN+SAO (7.70)</td>
<td>0.034 [ 0.001 – 0.106 ]</td>
</tr>
<tr>
<td>SAS+SAP+SAN (7.76)</td>
<td>0.001 [ 0.000 – 0.003 ]</td>
</tr>
<tr>
<td>SAP+SAN+SAO (7.92)</td>
<td>0.000 [ 0.000 – 0.011 ]</td>
</tr>
<tr>
<td>SAS+SAP+SAN+SAO (7.96)</td>
<td>0.047 [ 0.003 – 0.138 ]</td>
</tr>
<tr>
<td>Floating (6.9)</td>
<td>0.071 [ 0.004 – 0.264 ]</td>
</tr>
</tbody>
</table>
Application on Network:
San Francisco Bay Test Area

Bay Area Transportation Network
San Andreas Mw = 8.0 scenario

- SA 8.0 Event
- 2 Networks
  - 9 bridges
  - 16 bridges
- Location
- NEHRP Soil Types
- HAZUS Bridge Types

Map layers
- Highways
- Selection
- Closed Highways (623)

2006 IFED Lake Louise, Calgary
$\sigma$ of Aggregate Loss
(with Isotropic Ground Motion Correlation)

Ground Motion Correlation with Spatial Dependence

\[ \rho = \exp \left[ - \left( \frac{r_{ij}}{r_o} \right)^2 \right] \]

$\sigma_{Loss}$ vs. $\rho_{Damage}$ for 16 sites and 9 sites.

$\mathbf{I, I} :$ Simulation
$\mathbf{\bullet, \bullet} :$ Analytical

2nd IFED Lake Louise, Calgary

April 26-29, 2006
Aggregate Loss Distribution Model

Nearly Independent

Lognormal

Nearly Perfect Dependence

Heavy tails
Conclusions

• Bounds on Network Loss Uncertainty
  – CoV_L = [0.6, 1.5]

• Isotropic ground motion correlation model
  – CoV_L = [0.9, 1.3]

• Influence of Network Size
  – Loss dispersion is highly sensitive to correlation effects for Large Sized Networks
  – Computational effort ~O[n^2]

• Importance of ρ_G vs. ρ_D
  – cases (ρ_G = x, ρ_D = y) and (ρ_G = y, ρ_D = x)
  – approximately equal importance
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Thank You. Questions?