Two-stage Bayesian processes for spatially varying condition states

by

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Spatially distributed processes

Process with spatial characteristics

Process can be characterized by (unobservable) condition states $\theta$
Spatially distributed processes

Process with spatial characteristics

Set of $n$ related systems

Process can be characterized by (unobservable) condition states $\theta$

For each system we have response or observation $X_i$

$X_i$ depends on condition state $\theta_i$
Condition states

Features of condition states:
- represented by a random variable or a vector of random variables
- discrete (e.g. indicators) or continuous (e.g. this paper)
- cannot be “measured” or “observed”
- can be both descriptive and abstract in nature
- characterize:
  * “quality” of a process
  * measure of “past and present performance”
  * speed and extent of a process
  * “exposure”
One-stage Bayesian models

Using single CS

\[ X_1 \ldots X_n \]

System

\[ X_1 \ldots X_i \ldots X_n \]
One-stage Bayesian models

Using single CS – update the CS

System

\[ X_1 \quad \ldots \quad X_i \quad \ldots \quad X_n \]

\[ \theta \mid X \]

\[ \propto \prod_{i=1}^{n} f(x_i \mid \theta) \]
One-stage Bayesian models

Using independent CS

CS

$\theta_1$

$\theta_i$

$\theta_n$

System

$X_i$

$X_i$

$X_n$
Using independent CS – update the CS

One-stage Bayesian models
Two-stage Bayesian models

exchangeable CS

Hyper-parameters

CS

System

$\theta_1 \quad \ldots \quad \theta_i \quad \ldots \quad \theta_n$

$X_1 \quad \ldots \quad X_i \quad \ldots \quad X_n$
Two-stage Bayesian models

exchangeable CS – update CS $\theta_0$

Hyper-parameters

CS

System

$X_1$, ..., $X_0$, ..., $X_n$

$X^*$
Two-stage Bayesian models

exchangeable CS – update a new system with unknown CS $\theta_{n+1}$

Hyper-parameters $\alpha | X$

CS

System

$\theta_1$, $\ldots$, $\theta_i$, $\ldots$, $\theta_n$, $\theta_{n+1}$

$X_1$, $\ldots$, $X_i$, $\ldots$, $X_n$
Two-stage Bayesian models

correlated CS

Hyper-parameters

\[ \alpha \]

CS

\[ \theta_1 \quad \ldots \quad \theta_i \quad \ldots \quad \theta_n \]

System

\[ X_1 \quad \ldots \quad X_i \quad \ldots \quad X_n \]
Two-stage Bayesian models

correlated CS – update CS $\theta_0$

Hyper-parameters

System

$X_1$ ... $X_0$ ... $X_n$
Example
exchangeable CS

• Wind farm with 10 wind turbines
• Each turbine $i$ suffers a number $X_i$ of component failures or breakdown each year
• $X_i$ depends on a mean annual failure rate $\theta_i$
• After the first year of operation, $x_i$ is observed
• CS is updated by using one and two-stage Bayesian models
• $X_i$ and hyper-parameter are discrete
• CS is continuous
Example
exchangeable CS

Posterior condition states

- 1-stage Bayes, single CS
- 1-stage Bayes, multiple CS's
- 2-stage Bayes

Wind turbine

Expected CS

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7

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Example
exchangeable CS

Posterior Probability of exceeding a critical CS $\theta_{crit}=3.0$

- 1-stage Bayes, single CS
- 1-stage Bayes, multiple CS's
- 2-stage Bayes
One-stage Bayesian models:
• Either considers only a global CS for the entire process,
• Or, treats all systems as local and fully independent
→ are not an adequate technique for spatially distributed processes

Two-stage Bayesian models:
• independent condition states
• correlated CS, but computationally difficult
→ are more efficient, but computationally more challenging

Conclusion