System reliability analysis with spatial rainfall data: an example of rail network reliability subject to rainfall-induced landslides

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Abstract
This study concerns the susceptibility of part of the UK rail network to rainfall-induced landslides. A mathematical model is given for the failure of a series system, and a statistical model has been formulated for the joint distribution of rainfall at different points along the rail line. These have been used to investigate the response of embankment models along the rail line to current and future climate scenarios. It has been shown that embankment moisture profile at the end of the summer months has a critical effect on system stability, both in terms of expected failure timing, and probability of failure. Further, it is seen that with changing climate, the system stability will increase unless embankment material properties are degraded, while no change is seen in the contribution to failure of different parts of the rail line.

1 Introduction
The performance of many infrastructure systems depends on widely distributed rainfall, where extreme rainfall may induce failures at one or more locations. Engineering design problems of this nature include the design of flood defences and of reservoirs, and reliability analysis of transport networks and of urban drainage systems.

The current study concerns part of the UK rail network. An estimated 60% of the network’s 16000 route km is composed of earthworks. With climate change, extreme rainfall events are expected to increase (Ekström et al, 2005), leading to a concern that rainfall-induced landslides may increase. One part of the system where landslides are a particular problem is in the south west of England, where high winter rainfall rates, combined with plastic clays and nineteenth century embankment
construction methods, lead to intermittent imposition of speed restrictions caused by landslip and subsequent inspection and remediation work.

2 Mathematical model for system failure

In order to represent the system mathematically, a criterion is needed for failure of an individual slope. A simple criterion can be found in the factor of safety, which is the minimum ratio, calculated with regard to all plausible failure surfaces, between the resistance provided by the clay shear strength and the average shear stress along the assumed failure surface. A critical factor of safety \( s_c \) is defined, so a slope with a factor of safety of below \( s_c \) is assumed to have failed. In this case, for a series system composed of a number of slopes, system failure occurs when the slope with minimum factor of safety is below \( s_c \), here taken as 1. In mathematical terms, the probability of system failure, \( P_F \), for a system with \( n \) slopes can be written as follows:

\[
P_F = \int \left( \min_{i=1}^{n} \left( s_{r_i} (x,y) \right) < s_c \right) \rho_{XY} (r_1,\ldots,r_n) \, dx \, dy ,
\]

where \( I(.) \) is an indicator function, whose value is 1 if the argument is true, and 0 otherwise, \( s_{r_i} (x,y) \) is the factor of safety of slope \( i \), dependent on rainfall and material variables \( X \) and \( Y \), \( s_c \) is the critical factor of safety denoting limiting failure, and where \( \rho_{XY} (r_1,\ldots,r_n) \) is the joint probability density of the material properties and rainfall vectors, evaluated at locations \( r_i \) and \( r_j \).

Assuming independence between rainfall and material properties, this can be rewritten:

\[
P_F = \int \left( \min_{i=1}^{n} \left( s_{r_i} (x,y) \right) < s_c \right) \rho_X (r_1,\ldots,r_n) \, \rho_Y (r_1,\ldots,r_n) \, dx \, dy
\]

where \( \rho_X (r_1,\ldots,r_n) \) is the joint probability density of rainfall values at slope locations \( r_1,\ldots,r_n \), and \( \rho_Y (r_1,\ldots,r_n) \) is the joint probability density of material properties at locations \( r_1,\ldots,r_n \).

3 Rainfall model

Examination of failure information over a three year period has shown that failures are most likely to coincide with high levels of 10-day antecedent rainfall totals. Accordingly, all modelling has been undertaken with 10-day aggregated rainfall.

3.1 Characterisation of rainfall at a single site

The rainfall record at a single location may be viewed in two ways, firstly as a time series, and secondly as a distribution of rainfall intensities. Clearly, seasonality plays a significant part, so rainfall is analysed separately for each month.

Treating the rainfall record as a time series, examination shows very low correlation between adjacent 10-day time intervals, so rainfall intensities between successive time intervals have been taken as independent for all winter months.

The rainfall distribution is represented by a semi-empirical model. For all but the highest intensities, the rainfall probability distribution has been interpolated from the quantiles of the data record on which the model is to be based. Since limited rainfall records do not necessarily contain the most extreme values, the highest values are represented by a statistical distribution. Extremes of rainfall above a high threshold value at a particular location are represented by a Generalised Pareto distribution. This takes the form:
\[ P(Z > z) = \zeta \left[ 1 + \frac{\xi (z - u)}{\sigma} \right]^{-\frac{1}{\xi}}, \]

where \( z \) is the rainfall value, \( u \) is a threshold value, \( \zeta \) is the probability of the rainfall exceeding the threshold value \( u \), and \( \sigma \) and \( \xi \) are the scale and shape parameters respectively of the Generalised Pareto distribution. The final rainfall model should not be sensitive to the choice of threshold; this is ensured by examination of the change in mean exceedance level with threshold level (Coles, 2001). A consistent threshold level has been found to be the 95th centile of the data at a location.

### 3.2 Spatial characterisation of the historical rainfall record

In order to determine the simultaneous rainfall distribution at a number of locations along the rail line, historical daily records have been used. These are limited by the length of available records, and by the particular locations where data has been collected. Unbroken records exist for 17 gauges, for the years 1961 to 1994 inclusive. Their locations relative to the rail line are shown in Figure 1.

Correlation of the 10-day aggregated daily rainfall records at pairs of locations, after transformation to a Normal distribution, shows an approximately exponential decay with distance of the correlation coefficient (Figure 2). The dependence calculated is

\[ \rho = \exp (-0.0013 \times \text{distance}). \]

The spatial relationships found are conditional upon knowledge of the distributions at individual sites, which are provided here by simulated records.

### 3.3 Rainfall simulation for present and future conditions

Since this study concerns the predicted effect of climate change on the system reliability, rainfall records are required for both current and future climates. The UK Environment Agency weather generator, EARWIG, provides simulated rainfall and weather variables over a 5km grid throughout the UK (Kilsby et al, submitted). The rainfall record is provided by a stochastic rainfall generator, based on a 5-km grid of daily rainfall records throughout the UK, produced by interpolation from several thousand individual, incomplete data records (MetOffice, 2003). Future climate is represented in the Environment Agency weather generator using 50-km resolution climate predictions from the UK Climate Impacts Programme (Hulme et al 2000). These predictions indicate that summer rainfall will decrease, and winter rainfall will increase with future climate, but are of necessity crude, given the resolution. Statistical measures of the change in each 50-km grid square are applied to the current climate to produce simulated rainfall records for future climate scenarios. Other climate variables have been simulated at these locations, permitting a consistent, complete simulated daily weather record on the 5-km grid.

Simulation of the joint rainfall distribution is performed using a multivariate Normal distribution. Transformation to rainfall variables is performed by firstly taking the inverse Normal distribution, and then by interpolation between the quantiles of the simulated rainfall distribution at the appropriate location. Comparison between simulated and measured rainfall records for sites at different separation distances are shown in Figure 3.

### 4 Slope model

The rail line from London to Bristol runs through the Thames valley, over a series of highly plastic, over-consolidated clays. The rail line was built during the mid-19th century, and embankments were built from local material, hand extracted. Consolidation has come with time, and the resulting loss in height was made up by filling with extra ballast (Perry et al, 2003). The
effect on the embankment structure is that there are zones of sheared material with lower strength than the basic material (O’Brien et al, 2004), reducing both the bulk permeability and the effective cohesion of the embankment material.

As poor information exists about the exact materials of existing slopes on the rail line, a representative embankment is used for all slopes, with homogenous local clay fill. The slope is assumed to have a height of 6m, with slope angle of 1:2. In addition to a uniform clay fill, a 1m thick, high permeability surface ballast layer is assumed, together with a 0.5 m thick, high permeability soil layer on the slope (Anderson and Kneale, 1980).

4.1 Description of CHASM

Slope stability has been examined using the CHASM computer package (Anderson et al, 1988), which combines a slope hydrology model with a limit equilibrium stability model, permitting description of the response of the slope stability to the developing moisture regime during a period of rainfall.

The model is described in detail elsewhere (Wilkinson et al, 2002), but a cursory description is needed here for the purposes of this paper. The slope is modelled as a series of columns, subdivided into rectangular cells. Rainfall is allowed to infiltrate at the surface, and vertical flow within the unsaturated zone is governed by Richards’ equation. This is solved explicitly, with the unsaturated conductivity being defined by the Millington-Quirk procedure (Millington and Quirk, 1959). At the water table, lateral flow is modelled by Darcy’s equation for saturated flow, with the hydraulic potential defined by the gradient of the water table. The slope stability is calculated at each hour by Bishop’s simplified method (Bishop, 1955). This postulates a circular slip surface, and calculates both the shear stresses and the soil shear strength along the slip surface to provide a factor of safety. The minimum factor of safety is then found by comparing all slip surfaces centred on a prespecified grid. A schematic diagram of the CHASM slope stability model is shown in Figure 4.

4.2 Limitations of the CHASM model for the current study

The CHASM model permits the inclusion of evaporation, and the effects of vegetation in modifying the slope hydrology and strength (Wilkinson et al, 2002). The descriptions of the evaporation and vegetation are limited in their applicability to UK clay embankment slopes, since the model was designed to describe slope stability in a tropical environment, and a number of simplifications have consequently been made. In particular, evaporation is modelled as a sinusoid of constant amplitude. This is suitable for simulations of short periods, such as might be appropriate for investigating tropical storms, but less so for simulations representing several months. However, comparison of different evaporation levels has shown that the inclusion of an evaporation model has little effect.

Effective rainfall on the slope surface is represented by taking the difference between the rainfall and potential evapotranspiration. Mean effective rainfall between April and September in southern England is negative, so does not lend itself to simulation with CHASM over these months. Thus calculation of September moisture profiles cannot be undertaken, and it is necessary to infer profiles from the information in the literature (Ridley et al, 2004), and calculate conditional failure probabilities.
4.3 Slope material properties

The soil properties used by CHASM are as follows:

- Saturated permeability (m s\(^{-1}\))
- Saturated soil moisture content (m\(^3\) m\(^{-3}\))
- Saturated bulk density (unit weight) (kNm\(^{-3}\))
- Unsaturated bulk density (kNm\(^{-3}\))
- Effective cohesion (kPa)
- Effective friction angle (angle)
- Suction–moisture curve (-)

Empirical evidence from modelling with CHASM, and also the results of formal sensitivity analysis (Hamm et al, in press) indicate that the most critical of these properties are the permeability and the effective cohesion and friction angle. Accordingly, it is proposed to take these properties as variables in a system simulation, and to assume constant values for the other quantities.

O’Brien et al (2004) measured in-situ London clay embankment fill permeabilities ranging from 6*10\(^{-7}\) to 2*10\(^{-9}\) ms\(^{-1}\), with a median value of 3*10\(^{-8}\) ms\(^{-1}\). This can be interpreted as a Normal distribution of log\(_{10}\) permeability. Since no comparable measurements exist in the literature for in-situ embankment fill permeabilities in other clays, all permeabilities are taken as similar to those in London clay.

Cripps and Taylor (1981, 1986, and 1987) assembled results of a large number of laboratory and field measurements on the clays of southern England. A simple model can be inferred, that the mean effective friction angle of clay fill material varies linearly from 20° near London to 25° near Bristol. Effective cohesion is difficult to measure, so values are back-calculated from an assumption that the embankment is just stable when almost saturated; saturation is taken as that level when the embankment has experienced continuous rain for six months at 120% of the mean winter rainfall. In order to permit comparison between climate scenarios, a mean value of 4kNm\(^{-2}\) is taken for all locations. Variability of both cohesion and tangent friction angle are given by Rackwitz (2000), as is correlation between these properties.

4.4 Response of slope model to applied rainfall

Trials with the CHASM package have shown that with a low permeability fill, and high permeability ballast and soil, deep-seated failure occurs when the embankment is largely saturated, and is not sensitive to short-term temporal rainfall pattern. The finding that embankment failures are associated with saturation coincides with previous field studies (Ridley et al, 2004, Anderson and Kneale, 1980).

5 System simulation

The rail system is represented by a number of locations along the line, namely those locations where failures occurred between 2001 and 2004. Seven locations have been taken, with a maximum separation distance of 84km. These correspond to the locations marked on the main arm of the rail network shown in Figure 1, and on the southerly arm between Swindon and Bristol. At each location, a number of idealized slopes have been assumed. The mean geotechnical characteristics of the slopes are determined by location alone; variation is described in section 4.3. It is assumed that properties of the individual slopes are independent.
Starting from an initial moisture profile at the beginning of October, rainfall is applied in 10-day steps, for a six-month period. Uniform rainfall is applied to each slope for each 10-day period, and the rainfall intensity is taken from the multivariate statistical distribution described above, transformed to the appropriate rainfall distribution for the location, time and climate scenario. Three initial surface moisture profiles were chosen, comprising surface suction at the top of the embankment of 8m, 6m and 3m, and initial water table height of 3m below the embankment top at the centre. A further water table height of 4m below the embankment top was also used in conjunction with the driest of these surface moisture profiles.

Representation of each location as a single cross-section is clearly unrealistic as there are typically a number of cross-sections that may fail at a given earthwork. Choice of the number of representative cross-sections influences the failure probability, but without site-specific information on the spatial correlation of soil properties it is hard to justify any particular choice. Trials undertaken with different numbers of slopes in any given location show that 5 slopes for a location gives a possibility of a reasonably consistent probability of failure throughout the season.

6 Results of simulation

Figure 5 shows the probability of system failure by month, for the four different initial moisture profiles. It is clear that the initial slope moisture profile plays a very significant role in both the probability of failure, and also in the likely timing of failure; the higher the soil moisture at the beginning of October, the greater the probability of system failure, and the earlier the expected failure.

Figure 6 shows the total probability of system failure, for two initial moisture profiles, for current and climate and for four future climate scenarios. It can be seen that with future climates, and a given initial moisture profile, the probability of failure is predicted to decrease for all scenarios. This is because the probability of slope failure is dependent on the total net moisture input, as described in section 4.4, and although the winter rainfall is predicted to increase, so too is potential evaporation.

A number of caveats must be attached to this conclusion. Firstly, the comparison is made for constant initial moisture profiles. It is predicted that summer rainfall will decrease, and potential evapotranspiration will increase for all climate scenarios, and all locations in the vicinity of the rail line, meaning that according to these predictions embankments will be dryer in September. However, as summers become dryer, embankment material becomes desiccated, affecting the structure of the slope material, and making landslides more likely for a given rainfall sequence. Indeed, it has been noted in practice that after a hot dry summer, failures occurred at much lower rainfall levels than otherwise. The slope structure will change by increasing the permeability close to the slope surface, allowing moisture to infiltrate more rapidly. Repeated cycles of shrink-swell are also expected to lower the effective cohesion. Calculations have been performed to investigate by how much the mean cohesion would have to decrease in a future climate, in order for the probability of failure to remain the same. Table 1 shows the mean cohesion values for each future scenario, and for both initial moisture profiles examined. The change in cohesion required for the probability of system failure to increase, is seen to be small.

Examination of the contribution to system failure of different parts of the line did not show a significant change with future climate scenarios. This may be because the maximum separation distance between locations taken in this study is 84km, which is similar to the resolution of the climate prediction model, 50km. In order to examine whether this is an issue, a hypothetical system was tested, with slopes in Bristol, Swindon and London – a maximum separation distance
of 145km. Figure 7 shows the contribution to probability of failure of each location, both at the current climate, and for a medium high emissions scenario in 2080. It can be seen in both cases that there is a greater contribution from the western locations, as found in practice. However, although there is a slight difference in the slope angle of the relative contributions, this is not significant, indicating that there is not a great predicted contrast in the relative effect of climate change at both ends of the line. This is clearly an issue of the line used for the study; another conclusion could be drawn for a different line, where there is a greater contrast in the predicted future climate.

Conclusions
The current study has demonstrated a methodology to quantify the stability of a transport system subject to a distributed threat. This methodology can be used for other spatially distributed systems. The approach not only calculates the probability of failure, but also illustrates the contribution of different areas of the system, providing a basis for maintenance prioritisation.

However, in this instance, the conclusions are limited by the lack of detailed information about the material properties of the slopes in the system and by the inability to calculate end-of-summer slope moisture profiles. Equally, an understanding is needed of how desiccation alters the soil properties, affecting the behaviour of the embankment slopes.

The study of the impact of a randomly varying, spatially distributed loading such as rainfall on a system implies a linkage between spatial and temporal scales of variability. The response of the system to such a threat depends not only on the scale of the spatial variability of the loading, but also on the temporal system response. In this case, the scale of the system response has been shown to extend beyond the immediate period of antecedent rainfall to the climate of the preceding seasons. It is thus not surprising, perhaps, that such an extended temporal response demonstrates a lack of spatial sensitivity.

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References
Anderson MG, Kemp MJ and Lloyd DM Applications of soil water finite difference models to slope stability problems Proceedings of the 5th international symposium on landslides, Lausanne, July 1988, pp525-530
Bishop AW (1955) The use of the slip circle in the stability analysis of slopes Geotechnique 5 pp7-77
Cripps JC and Taylor RK (1986) Engineering characteristics of British over-consolidated clays and mudrocks I Tertiary deposits Engineering Geology 22, pp349-376
Cripps JC and Taylor RK (1987) Engineering characteristics of British over-consolidated clays and mudrocks II. Mesozoic deposits Engineering Geology 23, pp213-253
Hamm NAS, Hall JW, Anderson MG (in press) *Variance-based sensitivity analysis of the probability of hydrologically induced slope instability* Computers and Geosciences


### Table 1: Effective cohesion required to maintain system reliability in future climate

<table>
<thead>
<tr>
<th></th>
<th>Wetter initial moisture profile (kPa)</th>
<th>Dryer initial moisture profile (kPa)</th>
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<td>4.0</td>
</tr>
<tr>
<td>2080, low emissions</td>
<td>3.8</td>
<td>3.9</td>
</tr>
<tr>
<td>2080, medium low emissions</td>
<td>3.6</td>
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<tr>
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<td>2080, high emissions</td>
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Figure 1: Diagram of the rail line under study showing rainfall gauges and sites of recent landslip
Figure 2: Dependence of the correlation coefficient on separation of rain gauges

Figure 3: Comparison between simulated and actual rainfall records for different rain gauge separation distances

Figure 4: Schematic diagram of CHASM slope stability model
Figure 5: Probability of system failure by month

Figure 6: Changing probability of system failure with climate change

Note that the different values predicted for 2080 correspond to different emissions scenarios.

Figure 7: Contribution to system failure of different parts of the rail line