Effect of spatial distribution of flaws on glass ring-on-ring test strengths

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Abstract

Glass panes normally fail due to cracking that initiates and propagates from minute flaws that are distributed over the surface of the glass. The stress at which a glass pane will fail is influenced by the duration of loading (related to crack growth) and the size of the pane (related to the expected number of flaws). A probabilistic glass failure prediction model is often used to account for load-duration and size effects, and the parameters of the model can be determined from test results. The analysis of test results is generally complicated because failure does not necessarily occur at the location of the peak stress in a pane of glass. However, ring-on-ring tests produce a region of uniform bending and uniform biaxial stresses inside the inner ring, and therefore the failure stresses can be readily determined for any failure initiated within the ring. Nevertheless, a significant proportion of failures may occur outside the inner ring. This paper discusses ring-on-ring test results and the influence of failures that occur outside the inner ring.

1. Introduction

Until recently, the design of glass in buildings was normally carried out by architects and glaziers using empirical design rules developed by glass manufacturers. In recent years, however, the design of glass in buildings has attracted increasing attention from engineers, particularly with regard to the wind resistance of glass facades on high-rise buildings, and attempts have been made to develop design rules based on engineering analyses and theoretical models of glass strength.

It is normally assumed that glass panes fail due to cracking that initiates and propagates from minute flaws that are distributed over the surface of the glass. The stress at which a glass pane will fail is influenced by the duration of loading (related to crack growth) and the size of the pane (related to the likelihood of including a critical flaw). A probabilistic glass failure prediction model for glass strength is often used to account for load-duration and size effects, and the parameters of the model can be determined from test results.

The analysis of test results is generally complicated because failure does not necessarily occur at the location of the peak stress in a pane of glass. However, ring-on-ring tests produce a region of uniform bending and uniform biaxial stresses inside the inner ring, and therefore the failure stresses can be readily determined for any failure initiated within the ring. Nevertheless, a significant proportion of failures may occur outside the inner ring.

This paper presents an outline of the glass failure prediction model and discusses the use of ring-on-ring test results to determine the model parameters. In particular the paper presents new results concerning the influence of glass fractures that occur outside the inner ring. Reference is made to
some ring-on-ring test results for 4 mm and 6 mm glass, including a comparison of weathered and non-weathered samples.

2. The Glass Failure Prediction Model

Current developments in the theoretical modelling of glass strength are based mainly on a failure prediction model developed at Texas Tech University (Beason and Morgan, 1984). This model accounts for the geometrically non-linear load-response of thin glass plates (involving plate bending and membrane effects), the time dependence of glass strength (dependent on the applied stress-history), and the statistical variability of strengths.

The glass failure prediction model developed at Texas Tech University by Beason and others (Beason, 1980; Beason and Morgan, 1984) evolved from the fracture mechanics theory of glass strength proposed by Griffith (1920), the statistical strength theory of Weibull (1939), the ‘stress corrosion’ failure criterion of Charles (1958a, 1958b) and the load-duration theory of glass strength proposed by Brown (1972).

According to the fracture mechanics model of Griffith (1920) glass strength is determined by the nature of flaws (microcracks) in the glass. Thus the strength of glass is limited by the development of stress concentrations at crack tips, and fracture occurs instantaneously if the stress intensity factor reaches a critical value which is assumed to be a material property.

Weibull (1939) expressed the probability of rupture of a brittle body in terms of a risk factor $\beta$:

$$P_f = 1 - \exp[-\beta]$$

where $\beta$ represents the expected number of critical flaws. Particular forms of the Weibull risk factor $\beta$ were introduced, yielding the well-known Weibull distribution functions. Using a 2-parameter Weibull distribution, the probability of failure due to surface flaws can be expressed:

$$p_f = 1 - \exp(-\int_{\text{Area}} k \sigma^m dA)$$

where the integral represents the expected number of flaws that would fail at a stress less than or equal to $\sigma$. The stress $\sigma$ was taken to be the principal tensile stress for the case of uniaxial tension, or an equivalent stress for other stress states (accounting for the random orientation of flaws).

To explain the observed dependence of glass strength on load-duration, Charles (1958) postulated that flaws grow due to a process of 'stress corrosion' or 'fatigue'. Charles concluded that 'when a flaw grows to a critical dimension under the action of an atmosphere of water vapour, the rate of approach to this critical dimension is proportional to the sixteenth power of the flaw tip stress'.

Brown (1972) adapted Charles' stress corrosion model and postulated that 'the cumulative effect of arbitrary time-dependent stress $\sigma$ applied to a specimen until failure at time $t_f$ is constant'. Accordingly, the cumulative damage associated with failure was expressed in terms of 'Brown's Integral':

$$D_f = \int_0^{t_f} \sigma^n dt$$
where $D_f$ is constant (for a given specimen), and the exponent $n$ was taken to be the same as the exponent of 16 reported by Charles for stress corrosion of soda-lime glass. (Note, however, that the exponent $n$ operates on the nominal stress in Brown's Integral, whereas it operates on the crack tip stress according to Charles' stress corrosion theory.)

Brown used his stress-corrosion failure criterion to relate applied stresses to equivalent peak stresses $\sigma$ for a particular strain rate. Furthermore, Brown used a 2-parameter Weibull distribution to describe the distribution of equivalent peak stresses at failure (in effect assuming that $D_f$ is Weibull distributed). Thus for a glass plate of area $A$ loaded at a given rate to a peak stress $\sigma$:

$$P_f = 1 - \exp[-kA \sigma^m]$$

In particular, Brown examined 2 sets of test results for large square glass plates. One set of results was for plates of equal areas, progressively loaded to failure in about 30 seconds. The other set of results was for plates of various areas, progressively loaded to failure in about 15 minutes. Load-duration effects were normalized using Brown's Integral (Eqn 3), and a 'similarity' of stress distributions in the loaded plates was assumed so that the Weibull distribution parameter $m$ (Eqn 2) could be estimated. Thus Brown accounted for the observed effects of plate size and load duration, for plates with similar stress distributions.

The glass failure prediction model of Beason (1980) extended Brown’s load-duration model by incorporating the results of theoretical (geometrically non-linear) stress analyses of thin glass plates. To predict glass strength, Beason analysed the development of maximum principal tensile stresses over the surface of a loaded plate, then used Brown’s Integral to calculate ‘equivalent’ maximum principal tensile stresses $s_{60}$ of 60 seconds duration:

$$s_{60} = \left[ \int_0^t \sigma(t) \, dt / 60 \right]^{1/16}$$

To account for biaxial stress effects, these equivalent stresses were then ‘corrected’ depending on the ratio of the fully developed principal stresses (following the approach of Weibull, except that equivalent biaxial tensile stresses were used). Accordingly, the probability of failure of a glass plate was given by

$$p_f = 1 - \exp\left( -\int_{\text{Area}} ks_{60}^m dA \right)$$

where $s_{60}$ denotes the ‘corrected’ equivalent (biaxial) 60-second stress.

Thus, the glass failure prediction model extended the application of Brown's load-duration model of glass strength to include plates of any shape, accounting for dissimilar stress distributions through the use of advanced stress analysis techniques, accounting for geometric non-linearity.

The glass failure prediction model has been widely applied in the development of glass design standards, but some limitations have been noted in previous papers (Reid, 1984; 1991; 2000). Clearly, errors result when the non-linear stress history at a point is represented in terms of the history of maximum principal tensile stresses (of variable orientation) and the fully-developed minimum principal tensile stress (which is not necessarily representative of the developing stress field). Also the specification of an equivalent load level for an entire plate, based on an 'equivalent' stress at a single point (the failure origin), results in approximation errors for geometrically non-linear effects.
Furthermore, the model is fundamentally unrealistic because Brown's Integral implies that the probability of failure at or before time $t$ is effectively independent of the applied stress at that time (Reid, 1984). Also, it has been noted (Reid, 1991) that the glass failure prediction model implies that the surface flaw parameter $m$, which characterizes the 'shape' of the strength distribution, is independent of the stress history, whereas realistic qualitative crack growth models indicate that the shape of the strength distribution changes with time. Similarly, it has been shown (Reid, 2000) that the results obtained from the glass failure prediction model and ‘equivalent’ crack growth models can be very different (qualitatively and quantitatively), and hence the modelling uncertainty associated with the glass failure prediction model is large. Accordingly, the glass failure prediction model is best suited to representing glass strengths for specific test conditions, and caution should be exercised if extrapolating from one set of test conditions to another.

2. Ring-on-ring tests of glass strength

Many of the difficulties associated with the interpretation of glass test results (as described above) can be reduced by using a ring-on-ring test to produce a region of uniform biaxial bending stresses that are essentially proportional to the applied loads. A ring-on-ring test is conducted by placing a glass plate on a circular support ring of radius $r_s$ and then applying loads through a smaller concentric loading ring of radius $r_l$. Ring-on-ring tests produce uniform biaxial bending inside the inner (loading) ring, and the glass stresses are essentially proportional to the applied loads for moderate displacements (whereas the stresses in an edge-supported plate with pressure loading exhibit significant geometric non-linearity).

For a ring-on-ring test using a circular plate of radius $R$ and thickness $t$ (and with Poisson’s ratio $\nu$) the radial stresses $\sigma_r$ and circumferential stresses $\sigma_c$ within the inner loading ring are related to the applied load $P$ in accordance with the expression

\[
\sigma_r = \sigma_c = \frac{3P}{4\pi^2t}[2(1 + \nu)\ln\left(\frac{r_l}{r_s}\right) + (1 - \nu)\left(\frac{r_s^2 - r_l^2}{r_s^2} - \frac{r_l^2}{R^2}\right)]
\]  

(7)

Ring-on-ring testing has not been widely used, but in 2003 an ASTM Standard (ASTM C1499 Standard Test Method for Monotonic Equibiaxial Flexural Strength of Advanced Ceramics at Ambient Temperature) specified a ring-on-ring test method for the determination of the bending strength of ceramic plates. This standard specified that the loading ring and the support ring must be proportioned so that $0.2 \leq r_l/r_s \leq 0.5$. According to the standard, only failures inside the load ring should be used for the statistical analysis of strengths. To estimate the distribution of test strengths (for ramp loading), the standard recommends a minimum sample of 30 valid test results (involving failures inside the loading ring).

Similarly, the International Standards Organization has published a draft standard ISO/DIS 1288-5 (ISO, 2001) for ring-on-ring testing of glass, and the ISO standard requires $r_l/r_s= 0.2$ (corresponding to the lower limit specified in ASTM C1499).

The Standards and discussions of ring-on-ring testing suggest that the test strength statistics should be assessed on the basis of the test results obtained for fractures that originate within the region of uniform biaxial stresses inside the loading ring. Also, discussions of ring-on-ring testing seem to suggest that the proportion of failures outside the inner loading ring can be reduced to an
insignificant level by the application of appropriate experimental techniques (e.g., by minimising the friction at the loading and support rings).

However, regardless of the selected model of glass strength (stress-based or accounting for time-dependence or statistical effects), failures that originate outside the loading ring should be taken into account when assessing strength statistics. The influence that these failures (due to the spatial variability of glass strengths) have on ring-on-ring strength statistics is examined below, with reference to the stochastic glass failure prediction model described above.

3. Interpretation of ring-on-ring test results in accordance with the glass failure prediction model

In accordance with the glass failure prediction model, the strength statistics for ring-on-ring tests are influenced by not only the peak stresses developed inside the loading ring (as given by Equation 7) but also the stresses developed outside the loading ring. Radial and circumferential stresses outside the loading ring each decrease progressively as the distance from the loading ring increases, but the radial and circumferential stresses decrease at different rates as illustrated in Figure 1. The stresses shown in Figure 1 (normalised with respect to the stresses inside the loading ring) are for a circular glass disc with a radius $R$ of 60 mm, a support ring with a radius $r_s$ of 40 mm and a loading ring with a radius $r_l$ of 25 mm. This test geometry corresponds to some recent tests carried out at the University of Sydney (Khambatta and Leece, 2005), as discussed below.

Furthermore, according to the glass failure prediction model, the probability of failure is dependent on the duration of loading as expressed in Brown’s Integral (Equation 3) and the equivalent 60-second stress $s_{60}$ (Equation 5) which is related to a Weibull distribution of strengths (with a shape parameter $m$). It follows that for ring-on-ring tests conducted using a constant rate of loading (for ramp loading to failure), the distribution of peak loads (and peak stresses) would also be Weibull-distributed (with a shape parameter $m^*=m(n+1)/n$).

Hence, if ring-on-ring tests are conducted with a constant rate of loading (and constant stress rate) and the peak stresses at failure are denoted $\sigma_{\text{max}}$ (given by Equation 7), then the probability of failure inside the loading ring may be expressed

$$p_{\text{fi}} = 1 - \exp\left[-\left(\frac{\sigma_{\text{max}}}{\sigma_o}\right)^{m^*}\right] \quad (8)$$

where $\sigma_o$ is the characteristic stress for the area within the loading ring. Accordingly, the expected number of critical flaws (i.e., flaws that would fail at a stress less than or equal to $\sigma_{\text{max}}$) inside the loading ring is

$$n_i = \int \rho_o \left(\frac{\sigma_{\text{max}}}{\sigma_o}\right)^{m^*} dA \quad (9)$$

where $\rho_o$ is the density of flaws with strengths less than the characteristic stress $\sigma_o$ and the region of integration is the area inside the loading ring.

Similarly, the expected number of flaws outside the loading ring may be obtained by integration with respect to the stresses in that region. Considering the principal tensile stresses (the
circumferential stresses) shown in Figure 1, for the region between the load and support rings \((25 \leq r\ (mm) \leq 40)\) the tensile stresses are approximately

\[
\sigma(r) = (1.8333 - 0.03333 \cdot r)\sigma_{\text{max}}
\]  

and the expected number of critical flaws outside the loading ring (between the loading and support rings) is

\[
n_o = \int_{25}^{40} \rho_0 \left( \frac{\sigma(r)}{\sigma_0} \right)^n (2\pi r) dr
\]

Accordingly, the probability distributions of strengths within the loading ring and outside the loading ring (with negligible probability of failure outside the support ring) are given by the cumulative distribution functions \(F_i(\sigma_{\text{max}})\) and \(F_o(\sigma_{\text{max}})\), respectively:

\[
F_i(\sigma_{\text{max}}) = 1 - \exp[-n_i]
\]

\[
F_o(\sigma_{\text{max}}) = 1 - \exp[-n_o]
\]

Hence, the probabilities of failure inside the loading ring and outside the loading ring \((p_{f_i} \text{ and } p_{f_0}, \text{ respectively})\) are given by the convolution integrals:

\[
p_{f_i} = \int f_i(\sigma) [1 - F_o(\sigma)] d\sigma
\]

\[
p_{f_0} = \int f_o(\sigma) [1 - F_i(\sigma)] d\sigma
\]

where \(f_i(\sigma)\) and \(f_o(\sigma)\) are the probability density functions of strengths inside and outside the loading ring.

For example, results of 49 ring-on-ring tests conducted on 4 mm annealed glass using the test geometry corresponding to Figure 1, gave 30 failures originating inside the loading ring and the corresponding distribution of strengths for failures inside the loading ring is approximated by the Weibull distribution corresponding to Equation 8 with \(\sigma_0 = 89.5\ \text{MPa}\) and \(m = 3.9:\)

\[
p_{f_i} = 1 - \exp\left[ -\left( \frac{\sigma_{\text{max}}}{89.5} \right)^{3.9} \right]
\]

Taking this as an approximation to the distribution of glass strengths \(F_i(\sigma)\) inside the loading ring, the corresponding probabilities of failure inside and outside the loading ring \((p_{f_i} \text{ and } p_{f_0}, \text{ respectively})\) would be as shown in Figure 2.

However, Figure 2 also shows that the distribution of test strengths for failures inside the loading ring \(f_{i,\text{test}}(\sigma)\) is significantly different from the probability distribution of glass strengths inside the loading ring \(f_i(\sigma)\). Moreover, Figure 2 shows that the distribution of test strengths for failures
inside the loading ring $f_{i,test}(s)$ is the same as the distribution of test strengths for failures outside the loading ring $f_{o,test}(s)$ and these distributions are the same as the overall distribution of test strengths $f_{test}(s)$ (including all failures, regardless of the fracture origins). Hence:

$$F_{i,test}(\sigma_{max}) = F_{o,test}(\sigma_{max}) = 1 - \exp[-(n_i + n_o)]$$

(17)

Figure 2 also shows that, according to the Weibull model, the total probability of a test failure inside the loading ring is 0.64 and the complementary probability of failure outside the loading ring is 0.36 (corresponding to the ratio $n_i:n_o = 0.64:0.36$, for $m^* = 3.9$). This is in agreement with test results which gave a ratio $n_i:n_o = 0.61:0.39$.

Accordingly, the observed strength distributions are dependent on the effective stressed area that contains the expected number of critical flaws $n_{total} = n_i + n_o$ and therefore the characteristic strength $\sigma_o$ in accordance with Equation 8 is applicable to that effective stressed area (rather than the area inside the loading ring). Also, the proportion of failures in each region corresponds to the expected number of critical flaws in the region.

4. Anomalous ring-on ring test results

In the previous section, it was shown that the Weibull model matched observed probability distribution functions of glass strengths and matched the observed proportions of failures inside and outside the loading ring, for ring-on-ring tests on 4 mm annealed glass.

However, some results for ring-on-ring tests on glass have given anomalous results. Tests on 59 specimens of 6 mm annealed glass (using the same test configuration as shown in Figure 1) gave

$$p_{fi} = 1 - \exp[-\left(\frac{\sigma_{max}}{114}\right)^{5.95}]$$

(18)

for the overall probability distribution of strengths and the probability distributions of strengths inside and outside the loading ring. This is consistent with the Weibull model for strengths as discussed above. However, for $m^* = 5.95$, the Weibull model implies that the expected proportion of failures inside the loading ring is 72% (with 28% outside the loading ring), whereas the 59 tests produced only 25 failures (42%) inside the loading ring and 34 failures (58%) outside the loading ring.

This result indicates that the glass outside the loading ring failed at lower flexural tensile stresses than the glass inside the loading ring (and therefore had a lower characteristic strength $\sigma_o$). This might be related to the fact that the glass outside the loading ring must also resist shear stresses which are not present inside the loading ring.

Whatever the explanation for this anomalous behaviour, it is clear that ring-on-ring test results should be checked for consistency with regard to the proportions of failures inside and outside the loading rings.

Conclusions

Ring-on-ring test results clearly demonstrate that the spatial variability of glass strengths has a very significant effect on the effective strength of a glass plate. In the absence of spatial
variability, all ring-on-ring tests would produce failures inside the loading ring, in the region of greatest flexural stresses. However, in practice, a significant proportion of the glass failures produced in ring-on-ring tests originate outside the loading ring, in regions of relatively low flexural stresses.

It has been shown that ring-on-ring test results can be modelled using a probabilistic glass failure prediction model (based on a Weibull model of glass strengths). In particular, it has been shown that the model can be used to predict the proportions of failures that occur inside and outside the loading ring, and to predict the influence that spatial variability has on the test strength statistics obtained (considering failures originating both inside and outside the loading ring).

Furthermore, it has been shown that the corresponding probability distributions of test strengths are identical for failures originating inside or outside the loading ring (or the total combined results). This result applies even if the local strength distributions inside and outside the loading ring are not the same (provided the Weibull shape parameter is constant). However, if the local strength distributions inside and outside the loading ring are not the same, the proportions of failures inside and outside the loading ring will change.

It has been shown that the glass failure prediction model successfully predicts the proportions of failures that were obtained inside and outside the loading ring for a set of tests on 4 mm annealed glass (assuming that the glass strength was identically distributed in the regions inside and outside the loading ring). However, a set of ring-on-ring tests on 6 mm annealed glass has produced results that can be explained by the glass failure prediction model only if it is assumed that the flexural strength outside the loading ring is less than the strength inside the ring. This might indicate that the effective flexural strength outside the loading ring is reduced by the presence of shear stresses (which are larger for the 6 mm glass than the 4 mm glass). If this is true, then the current theoretical basis for the design of glass (based on flexural stresses only) may be fundamentally flawed.

References


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Reid, S.G. (2000), Model Errors in the Failure Prediction Model of Glass Strength, Eighth ASCE Joint Specialty Conference on Probabilistic Mechanics and Structural Reliability, University of

Figure 1: axisymmetric stresses for ring-on-ring test
Figure 2: Probability distribution functions of ring-on-ring glass strengths